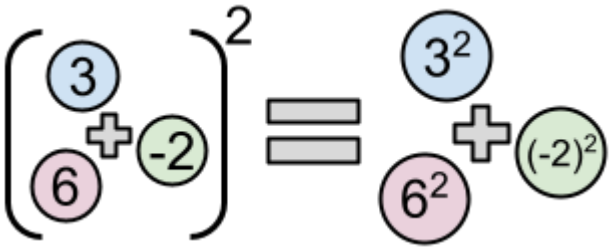
Code Jam 2022 - Round 1C

Squary

Problem

Addition and squaring do not commute. That is, the square of the sum of all elements of a list of integers is not necessarily equal to the sum of the squares of those same elements. However, this is true for some lists; one example is $[3, -2, 6]$, because $(3 + (-2) + 6)^2 = 49 = 3^2 + (-2)^2 + 6^2$. Let us call these lists *squary*.



Given a (not necessarily squary) list of relatively small integers, we want to know whether it is possible to add at least 1 and at most $\mathbf{K}$ more elements such that the final list is squary. Each added element must be an integer between -10^{18} and 10^{18}, inclusive, and these do not have to be distinct from each other or from the initial list's elements.

Input

The first line of the input gives the number of test cases, $\mathbf{T}$. $\mathbf{T}$ test cases follow. Each test case is described in two lines. The first line contains two integers $\mathbf{N}$ and $\mathbf{K}$, the number of elements of the initial list and the maximum number of elements you may add, respectively. The second line contains $\mathbf{N}$ integers $\mathbf{E_1}, \mathbf{E_2}, \ldots, \mathbf{E_N}$, representing the $\mathbf{N}$ elements of the initial list.

Output

For each test case, output one line containing `Case #`x`:` y, where x is the test case number (starting from 1). If it is possible to add at least 1 and at most $\mathbf{K}$ elements (each an integer between -10^{18} and 10^{18}, inclusive) to the initial list such that the square of the sum of its elements equals the sum of the squares of its elements, y should be $z_1\ z_2\ \ldots\ z_r$, where $1 \leq r \leq \mathbf{K}$ and the z_i values are the additional elements. If there is no way to accomplish this, y should be `IMPOSSIBLE`.

Limits

Memory limit: 1 GB.
$1 \leq \mathbf{T} \leq 100$.
$1 \leq \mathbf{N} \leq 1000$.
$-1000 \leq \mathbf{E_i} \leq 1000$, for all i.

Test Set 1 (Visible Verdict)

Time limit: 5 seconds.
$\mathbf{K} = 1$.

Test Set 2 (Visible Verdict)

Time limit: 10 seconds.
$2 \leq \mathbf{K} \leq 1000$.

Sample

Note: there are additional samples that are not run on submissions down below.

Sample Input

```
4
2 1
-2 6
2 1
-10 10
1 1
0
3 1
2 -2 2
```

Sample Output

```
Case #1: 3
Case #2: IMPOSSIBLE
Case #3: -10000000000000000000
Case #4: 2
```

In Sample Case #1, we can end up with the example list given in the problem statement.

In Sample Case #2, we have to add exactly one element. If we call that element x, the sum of the entire list is x and its square is x^2. The sum of the squares of all elements, on the other hand, is $x^2 + 10^2 + (-10)^2 = x^2 + 200 \neq x^2$, so the case is impossible.

In Sample Case #3, any integer in the $[-10^{18}, 10^{18}]$ range is a valid answer.

In Sample Case #4, notice that the input might contain duplicate elements, and that it is valid to create even more duplicates with the elements you choose to add.

Additional Sample - Test Set 2

The following additional sample fits the limits of Test Set 2. It will not be run against your submitted solutions.

Sample Input

```
3
3 10
-2 3 6
6 2
-2 2 1 -2 4 -1
1 12
-5
```

Sample Output

```
Case #1: 0
Case #2: -1 15
Case #3: 1 1 1 1 1 1 1 1 1 1 1
```

In Case #1 of the additional samples, we are given the example list from the problem statement, which is already squary, but we need to add at least one element to it. Adding a 0 keeps the list squary.

In Case #3 of the additional samples, we present one of multiple possible valid answers. Notice that it is permissible to add fewer than $\mathbf{K}$ elements; here $\mathbf{K}$ is 12 but we have only added 11 elements.